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SUMMARY

A creep damage accumulation model is presented that makes use of the Kachanov damage rate concept with a provision accounting for damage that results from a variable stress history. This is accomplished through the introduction of an additional term in the Kachanov rate equation that is linear in the stress rate. Specification of the material functions and parameters in the model requires two types of tests constituting a data base:

- (1) standard constant-stress creep rupture tests, and
- (2) a sequence of two-step stress creep rupture tests

INTRODUCTION

One of the primary failure modes considered in the ASME Code Case N-47 (Ref. 1) for structural components in elevated-temperature service is that of creep rupture under quasi-steady, long-term loading. The Code Case specifies that predictions of time to failure under such loading conditions should be based on the "time-fraction law," originally proposed by Robinson (Ref. 2),

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 1 \quad (1)$$

in which t_f denotes the time to failure and $t_R(\sigma)$ represents the "creep-rupture curve" determined from uniaxial tensile creep tests at constant stress.

Several authors (e.g., Refs. 3 to 5) have pointed out that Eq. (1) is not strictly satisfied for many structural alloys under variable stress conditions. For example, it is often observed that

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} < 1 \quad (2)$$

in uniaxial two-step creep rupture tests involving a step-up in tensile stress and

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} > 1 \quad (3)$$

for a stress down-step*.

Here, a creep damage model is proposed similar to that originally proposed by Kachanov (Ref. 6) – and employed or extended by numerous workers (e.g., Refs. 7 to 9) – but which includes an additional term depending linearly on the stress rate. This results in a time-independent contribution to the predicted damage arising from variations in stress in an already creep damaged material. The model is stated here for isothermal and noncyclic conditions; extensions to nonisothermal conditions and to reverse stressing will be subjects of continued research.

The proposed damage rate equation is shown to be equivalent to a damage law of the form

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 1 + \alpha \quad (4)$$

in which t_f and $t_R(\sigma)$ have the same meanings as in Eq. (1) and α is a functional of the stress history $\sigma(t)$. For some stress histories α is negative and Eq. (4) predicts a comparatively shorter time to failure than Eq. (1); in other cases α is positive and Eq. (4) predicts a longer time to failure.

*Notably, this effect of stress sequencing is opposite to that ordinarily observed in fatigue testing.

The implementation of the model requires a data base made up of two types of tests:

- (1) standard constant-stress creep rupture tests, and
- (2) variable (two-step) stress creep rupture tests.

A unique feature here is that the variable stress experiments are used not solely as tests to furnish evidence for verification of the model, as is commonly the case, but instead they make up an essential part of the data base for establishing the functional forms and parameters of the model.

As a background for the development of the proposed model, the connection between the Kachanov damage model and the time-fraction law, Eq. (1), will first be demonstrated. The proposed model will then be stated along with a discussion of the testing required for its specification.

THE KACHANOV MODEL AND THE TIME-FRACTION LAW

The Kachanov damage model, giving the rate of degradation of a material under creep at elevated temperature, is frequently stated in the separable form

$$\dot{\psi} = - \frac{f(\sigma)}{g'(\psi)} \quad (5)$$

in which ψ is the material continuity and σ is the applied stress. For a material completely intact $\psi = 1$, and for complete destruction $\psi = 0$.

The rupture life t_R under constant stress is found by integration of Eq. (5) and is

$$t_R(\sigma) = \frac{g(1) - g(0)}{f(\sigma)} = \frac{\text{const.}}{f(\sigma)} \quad (6)$$

In the special case where $f = C\sigma^n$, Eq. (6) takes the familiar form

$$t_R(\sigma) = \frac{\text{const.}}{\sigma^n} \quad (7)$$

The failure time t_f under an arbitrarily varying stress is found from Eqs. (5) and (6) as

$$g(1) - g(0) = \int_0^{t_f} \frac{g(1) - g(0)}{t_R(\sigma)} dt \quad (8)$$

or

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 1 \quad (9)$$

which is identical to the time-fraction law, Eq. (1). Whatever the functions f and g , Eq. (5) demands the satisfaction of Eq. (1) at failure. The Kachanov creep damage model, expressed in the separable form of Eq. (5), cannot lead to the conditions stated in Eqs. (2) or (3).

THE PROPOSED DAMAGE MODEL

Here, a damage rate equation similar in spirit to that of Eq. (5) is introduced that contains an additional term proportional to the stress rate $\dot{\sigma}$. That is, we propose the rate equation

$$\dot{\psi} = h(\sigma, \psi) \dot{\sigma} - C \frac{\sigma^n}{\psi^m} \quad (10)$$

in which the second term is taken to be a special form of the Kachanov equation, Eq. (5), whereas the first term is taken linear in the stress rate $\dot{\sigma}$. The first term represents a time-independent contribution which arises from changes in applied stress; the second term represents the usual time-dependent contribution as proposed by Kachanov. The constants C , n , and m and the function h are determined by experiment as described below.

Equation (10) is consistent with the view that creep damage occurs as the result of the creation of interior voids or cavities. In the context of step-wise creep tests, the first term in Eq. (10) allows for either further damage or healing of prior damage as a consequence of changes in stress. This can be thought of as resulting from further widening or partial closing of existing

voids depending on the direction of the stress change. So that stress changes do not effect the yet undamaged material, it is reasonable to restrict the function h such that

$$h(\sigma, 1) = 0 \quad (11)$$

With this restriction, it is seen that in ordinary creep rupture tests, in which an undamaged specimen is brought relatively abruptly to a constant stress and held (i.e., $\dot{\sigma} = 0$), there is no contribution from the first term in Eq. (10) throughout the test and we can write

$$\int_1^0 \psi^m d\psi = - \int_0^{t_R} C \sigma^n dt \quad (12)$$

or

$$t_R(\sigma) = \frac{1}{C(m+1)\sigma^n} \quad (13)$$

Optimal values of $1/[C(m+1)]$ and n can thus be determined using Eq. (13) and data pairs in the form (t_R, σ) from standard creep rupture tests.

The variation of the material continuity ψ in creep rupture tests is likewise found by integrating Eq. (10) with the first term absent. Thus

$$\psi = \left(1 - \int_0^t \frac{dt}{t_R(\sigma)} \right)^{1/(m+1)} \quad (14)$$

Only for the very special case $m = 0$, corresponding to the second term in Eq. (10) being independent of ψ , is the diminuation of material continuity (or the accumulation of damage $D = 1 - \psi$) linear in time. With $m > 0$, ψ varies nonlinearly in time, slowly at first but at a higher rate as internal damage occurs.

Hypothetical trajectories of two creep rupture tests at stress levels of σ_1 and σ_2 are shown as the solid lines in the $\left(\psi, \sigma, \int \frac{dt}{t_R} \right)$ plot of Fig. (1).

Each curve originates at the intersection of $\psi = 1$ and $\int \frac{dt}{t_R} = 0$ and each terminates, at failure, in the plane $\psi = 0$. The locus of rupture points in $\psi = 0$ is defined by $\int \frac{dt}{t_R} = 1$.

We now assume that the stress dependence of the minimum creep rate $\dot{\epsilon}_m$ in constant-stress creep tests can be adequately represented by the Norton law (Ref. 10)

$$\dot{\epsilon}_m = A\sigma^N \quad (15)$$

where A and N are known constants. The subsequent development is not dependent on the particular choice of the Norton law; it has been chosen because it has wide application and is simple. We further assume that the subsequent acceleration of creep in such tests is due to internal damage accumulation* and that, following Kachanov, Eq. (15) remains valid with σ replaced by the "effective" stress σ/ψ , i.e., we take

$$\dot{\epsilon} = A \left(\frac{\sigma}{\psi} \right)^N \quad (16)$$

Dividing Eq. (15) by Eq. (16) and solving for ψ , we get

$$= \left(\frac{\dot{\epsilon}_m}{\dot{\epsilon}} \right)^{1/N} \quad (17)$$

indicating that we can assign values of ψ along a given creep-rupture curve.

The minimum creep rate $\dot{\epsilon}_m$ can be readily identified, and $\dot{\epsilon}$ can be determined at each instant of time along the curve.

Eliminating ψ from Eqs. (14) and (17) we get:

*If, as is usually the case, the creep rupture tests are conducted under constant load rather than constant stress, the contribution of the creep acceleration related to geometry change must be accounted for.

$$\left(\frac{\dot{\epsilon}_m}{\dot{\epsilon}}\right) = \left(1 - \int_0^t \frac{dt}{t_R(\sigma)}\right)^{N/(m+1)} \quad (18)$$

As $(\dot{\epsilon}_m/\dot{\epsilon})$ is measurable along a given creep curve, we can generate data sets in the form $(\dot{\epsilon}_m/\dot{\epsilon}, \sigma, t)$ directly from creep-rupture tests and use these with Eq. (18) to obtain optimal values of $N/(m+1)$. With N known, this establishes m ; with m and $1/[C(m+1)]$ in Eq. (13) known, C is determined. Thus, n , m , and C are known and the second term in Eq. (10) is fully specified from creep-rupture data alone.

Before proceeding with the determination of $h(\sigma, \psi)$ in Eq. (10), we consider the predicted response of a two-step stress test when $h \equiv 0$ and Eq. (10) is of the classical Kachanov form. In this case, the image of a typical two-step creep rupture test is shown as OABC in Fig. 1. As the abrupt step from σ_1 to σ_2 occurs (AB), ψ remains constant. Note that the projections of the step test OABC as well as those of the constant stress tests at σ_1 and σ_2 are identical in the $\left(\psi, \int \frac{dt}{t_R}\right)$ plane. Moreover, for each test, constant or variable stress,

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 1 \quad (19)$$

at failure.

Next, we consider stepped creep rupture tests designed to establish the form of the function $h(\sigma, \psi)$. The image, predicted by Eq. (10), of a test involving a step increase from σ_1 to σ_2 is indicated as OABC in Fig. 2. The stress is held constant at σ_1 in $0 < t < t_A$ (t_A being the time corresponding to point A). At $t = t_A$ the stress is abruptly increased to σ_2 (along AB) and again held constant. We suppose that creep rupture occurs at the time $t = t_C$ (corresponding to the point C in Fig. 2). Here, ψ is re-

presented as decreasing (indicating further damage) as the stress is increased abruptly to σ_2 . Correspondingly,

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} < 1 \quad (20)$$

at the failure time $t_f = t_c$.

The case where a step down in stress is made from σ_2 to σ_1 at $t = t_A$ is depicted in Fig. 3. Here, ψ is represented as increasing (indicating partial healing) as the stress is decreased and,

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} > 1 \quad (21)$$

at the failure time $t_f = t_c$.

Over the intervals when the state point follows the path segment OA or BC in Figs. 2 or 3, $\dot{\sigma} = 0$ and the first term in Eq. (10) does not contribute to $\dot{\psi}$. During initial load-up $\psi = 1$, corresponding to the undamaged material condition, and likewise the first term does not contribute, in accordance with Eq. (11). However, as the abrupt stress change AB is made, the material has incurred prior damage and, as the time interval of loading is short, the first term in Eq. (10) dominates; the continuity ψ , then increases or decreases, depending on the nature of $h(\sigma, \psi)$ and the stress history.

In light of the foregoing discussion, ψ_A , (e.g., in Fig. 2), is found by integrating Eq. (10), including only the second term, thus

$$\int_1^{\psi_A} \psi^m d\psi = - \int_0^{t_A} C \sigma_1^n dt \quad (22)$$

or

$$\psi_A = \left(1 - \frac{t_A}{t_R(\sigma_1)}\right)^{1/(m+1)} \quad (23)$$

Similarly, ψ_B is found by

$$\int_{\psi_B}^0 \psi^m d\psi = - \int_{t_A}^{t_C} C\sigma_2^n dt \quad (24)$$

or

$$\psi_B = \left(\frac{t_C - t_A}{t_R(\sigma_2)}\right)^{1/(m+1)} \quad (25)$$

The change in ψ during the abrupt step change from σ_1 to σ_2 is thus the difference of Eq. (23) and Eq. (25). Equivalently, we know the points A' (σ_1, ψ_A) and B' (σ_2, ψ_B) as projected into the σ, ψ plane (Fig. 4).

If we were to perform a sequence of step tests of this kind, each starting at $\sigma = \sigma_1$ and with a step (up or down) at $t = t_A$, we would map out a curve in the σ, ψ plane containing A' and B'. Further, if we conducted several sequences of step tests commencing at other stress levels and with stress changes at other times, we would eventually map out an entire family of curves in a region of the σ, ψ plane (Fig. 5). Figure 5 is schematic and real data would likely show considerable scatter, nevertheless, it is reasonable to expect that the underlying trends can be satisfactorily represented by a family of curves

$$p(\sigma, \psi) = \text{const.} \quad (26)$$

which fits the data in a least-squares sense.

As the abrupt stress change occurs in any of the step-stress tests, the state point moves along one of the curves represented by Eq. (26). As discussed earlier, the first term in the rate Eq. (10) governs, i.e., we have

$$\dot{\psi} = h(\sigma, \psi) \dot{\sigma} \quad (27)$$

We can eliminate time in Eq. (27) and write

$$\frac{d\psi}{d\sigma} = h(\sigma, \psi) \quad (28)$$

Now, we want the experimental curves represented in Eq. (26) to be the integral-curves of Eq. (28), that is to say, we want the function $h(\sigma, \psi)$ to be given by

$$\frac{d\psi}{d\sigma} = - \left(\frac{\frac{\partial p}{\partial \sigma}}{\frac{\partial p}{\partial \psi}} \right) = h(\sigma, \psi) \quad (29)$$

With $p(\sigma, \psi)$ specified, and thus $\partial p / \partial \sigma$ and $\partial p / \partial \psi$ known, we then have an explicit form for $h(\sigma, \psi)$.

This completes the specification of the rate equation, Eq. (10), and we can supposedly use it to predict the time to creep failure under an arbitrary stress history $\sigma(t)$. In general, this is done by numerically integrating Eq. (10) to determine the time at which $\psi > 0$.

The approach followed here in determining the function $h(\sigma, \psi)$ on the basis of two-step creep rupture tests is analogous to that followed by Leckie and Ponter (Ref. 11) in obtaining representations of creep deformation under variable stress conditions. In their case, stepped creep tests provided a description of how to move from one constant stress creep curve to another as the stress varied. Here, stepped creep rupture tests similarly provide a description of how to move from one constant stress rupture curve, in the $(\psi, \sigma, \int \frac{dt}{t_R})$ space of Fig. 2 or 3, to another as the stress is varied.

The success of the model proposed here depends on the ability to conduct a sequence of accurate and repeatable two-step stress rupture tests. Assuming this can be done, the model must, at the very least, give back accurate predictions of creep rupture under stepped loading, since such tests comprise its data base. In principle, the model should also provide good failure predic-

tions under general (tensile) variable stress conditions, although the proof of this must obviously come from experiment.

THE PROPOSED MODEL AND THE TIME-FRACTION LAW

To demonstrate that Eq. (10) is equivalent to the form shown in Eq. (4), we integrate Eq. (10) for an arbitrary stress history $\sigma(t)$ as follows:

$$\int_1^0 \psi^m d\psi = \int_0^{t_f} \psi^m h(\sigma, \psi) \dot{\sigma} dt - \int_0^{t_f} C \sigma^n dt \quad (30)$$

Using Eq. (13) and calling $\sigma_i = \sigma(0)$ and $\sigma_f = \sigma(t_f)$, we get

$$1 = - (m + 1) \int_{\sigma_i}^{\sigma_f} \psi^m h(\sigma, \psi) d\sigma + \int_0^{t_f} \frac{dt}{t_R(\sigma)} \quad (31)$$

or

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 1 + \alpha \quad (32)$$

in which

$$\alpha = (m + 1) \int_{\sigma_i}^{\sigma_f} \psi^m h(\sigma, \psi) d\sigma \quad (33)$$

Note that the integral in Eq. (33) is path dependent so that the value of α depends on the stress history $\sigma(t)$.

SOME HYPOTHETICAL FORMS OF $p(\sigma, \psi)$

The key to the behavior under variable stress lies in the form of $p(\sigma, \psi)$ in Eq. (26). Experiments are likely to show that the p curves of Fig. 5 are characteristically different in different regions of the σ, ψ plane, reflecting distinct rupture mechanisms. In any case, if a sufficient number of variable stress rupture tests are conducted spanning the relevant regions of the σ, ψ plane, this information should be inherently built-in to the model.

In the absence of sufficient variable stress tests, it is instructive to hypothesize some simple forms of $p(\sigma, \psi)$ and examine the consequences on the predicted creep rupture behavior.

In regions of the σ, ψ plane where Eq. (26) represents a family of horizontal lines (Fig. 6), $h = 0$ in Eq. (10) and the model is equivalent to the usual time-fraction law expressed in Eq. (1).

An interesting case is provided by the family of curves

$$p(\sigma, \psi) = \frac{1 - \psi^s}{\sigma} = \frac{1}{\sigma_0} = \text{const.} \quad (34)$$

in which s and σ_0 are constants. We then have

$$\frac{\partial p}{\partial \sigma} = - \frac{1 - \psi^s}{\sigma^2} \quad (35)$$

and

$$\frac{\partial p}{\partial \psi} = - \frac{s}{\sigma} \psi^{s-1} \quad (36)$$

so that

$$h(\sigma, \psi) = - \frac{1}{s\sigma} \left(\frac{1 - \psi^s}{\psi^{s-1}} \right) \quad (37)$$

Note that, in agreement with Eq. (11),

$$h(\sigma, 1) = 0 \quad (38)$$

Equation (10) thus becomes

$$\dot{\psi} = - \frac{1}{s\sigma} \left(\frac{1 - \psi^s}{\psi^{s-1}} \right) \sigma - C \frac{\sigma^n}{\psi^m} \quad (39)$$

The family of curves corresponding to $s = 2$ and several values of σ_0 in Eq. (34) is shown in Fig. 7. Using these and appropriate values for m , σ_i and σ_f , we can calculate the time-fraction integral, Eq. (32), for any given stress history. Consider, for example, the two-step tests depicted in Fig. 8; the first involves a step-up in stress from $\sigma_i = 140$ to $\sigma_f = 175$ MPa (Fig.

8(a)) and the second a step-down from $\sigma_i = 140$ to $\sigma_f = 105$ MPa (Fig. 8(b)). With m taken as $m = 1.5$, values of the time-fraction integral have been calculated and are given in Tables I and II for the stress histories of Figs. 8(a) and (b), respectively. The quantity β in the tables denotes the fraction of the rupture life $t_R(\sigma_i) = t_R(140)$ at which the step up (or down) in stress is made. When the step is made relatively early in life (e.g., $\beta = 0.2$), the value of the integral at failure is not very different from unity; when the step is made relatively late in life (e.g., $\beta = 0.8$), the integral may differ substantially from unity.

The point 0 in Fig. 7 represents the point in the σ, ψ plane from which the abrupt stress step is made corresponding to $\beta = 0.2$ in Table I or II. As the stress is increased from 140 to 175 MPa, the state point moves from 0 to a. As the stress is decreased to 105 MPa, the point moves to b. Only slight additional damage or healing is incurred during the stress change. The predicted values for the time-fraction integral at failure are

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 0.95 \quad (40)$$

for the step-up in stress, and

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 1.05 \quad (41)$$

for the down-step.

The point 0' in Fig. 7 indicates the point from which a step is made with $\beta = 0.8$. The material has now incurred considerable prior creep damage, and a step-up in stress to 175 MPa moves the state point to a', indicating substantial additional damage as the stress is increased. The value of the time-fraction at failure in this case is

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 0.85 \quad (42)$$

Note that here, following the expenditure of 0.8 of the rupture life at $\sigma = 140$ MPa, a stress increase to approximately 193 MPa or greater causes failure immediately as the stress is changed, corresponding to $\psi \rightarrow 0$.

A step down in stress from $0'$ moves the state point to b' providing partial healing, and the integral at failure is

$$\int_0^{t_f} \frac{dt}{t_R(\sigma)} = 1.18 \quad (43)$$

Additional forms of $p(\sigma, \psi)$ can be hypothesized and calculations made similar to those above, however, such exercises would probably not be fruitful at present. It is preferable to wait until sufficient experimental data can be generated and the actual forms of $p(\sigma, \psi)$ and $h(\sigma, \psi)$ determined.

It is possible that a single family of curves in the σ, ψ plane is not sufficient to represent the effect of both stress increases and stress decreases on the creep damaged material. If this were the case, the same principles and procedures outlined above are still applicable with the following modifications and reinterpretations. The damage rate equation, Eq. (10), could then be modified to

$$\dot{\psi} = h_1(\sigma, \psi) \langle \dot{\sigma} \rangle + h_2(\sigma, \psi) \langle -\dot{\sigma} \rangle - C \frac{\sigma^n}{\psi^m} \quad (44)$$

with

$$\langle x \rangle = x; \quad x > 0$$

and

$$\langle x \rangle = 0; \quad x \leq 0$$

in which

$$h_1(\sigma, \psi) = - \left(\frac{\frac{\partial p_1}{\partial \sigma}}{\frac{\partial p_1}{\partial \psi}} \right) \quad (45)$$

and

$$h_2(\sigma, \psi) = - \left(\frac{\frac{\partial p_2}{\partial \sigma}}{\frac{\partial p_2}{\partial \psi}} \right) \quad (46)$$

The family of curves represented by $p_1(\sigma, \psi) = \text{const.}$ in the σ, ψ plane would be determined from a sequence of stepwise stress rupture tests involving a step-up in stress and the curves $p_2(\sigma, \psi) = \text{const.}$ from a sequence with a step-down in stress.

CONCLUSIONS

The proposed damage model is limited in applicability to a narrow range of conditions, i.e., uniaxial, isothermal, variable tensile stress histories. Nevertheless, the existing technology supporting the design of elevated temperature components (e.g., the guidance provided by ASME code case N-47) does not provide an adequate methodology for accurately predicting creep rupture even under these restricted conditions.

The attempt here is to provide a simple model which has the potential for predicting failure in, perhaps, the most fundamental of problems of high-temperature design, creep rupture under quasi-steady, long-term loading. Once these predictions can be made with reasonable consistency and accuracy, only then does it seem appropriate to concentrate on further complexities such as cyclic stressing, multiaxiality, creep-fatigue interactions, etc.

As the present creep damage model is based on both standard creep rupture tests and stepwise stress rupture tests, it is expected that it should provide accurate predictions under general (tensile) variable stress conditions. This, of course, must ultimately be demonstrated by experiment.

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TABLE I

β	$\int \frac{dt}{t_R}$
0.2	0.951
.3	0.929
.4	0.907
.5	0.887
.6	0.870
.7	0.857
.8	0.851

TABLE II

β	$\int \frac{dt}{t_R}$
0.2	1.049
.3	1.073
.4	1.096
.5	1.178
.6	1.139
.7	1.159
.8	1.176

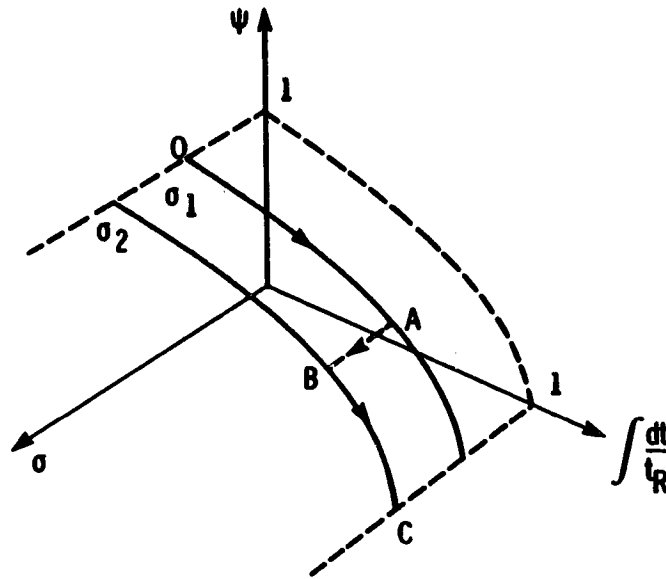


Figure 1. - Representation of constant stress creep rupture tests in $(\psi, \sigma, \int \frac{dt}{t_R})$ space.

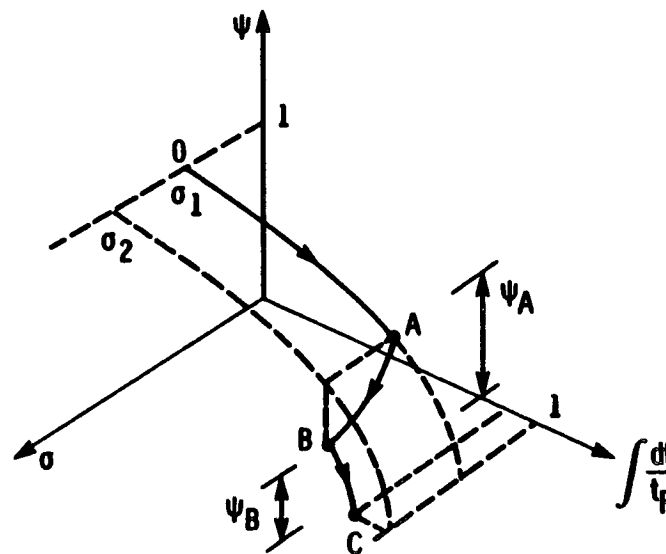


Figure 2. - Representation of step-up creep rupture test in $(\psi, \sigma, \int \frac{dt}{t_R})$ space.

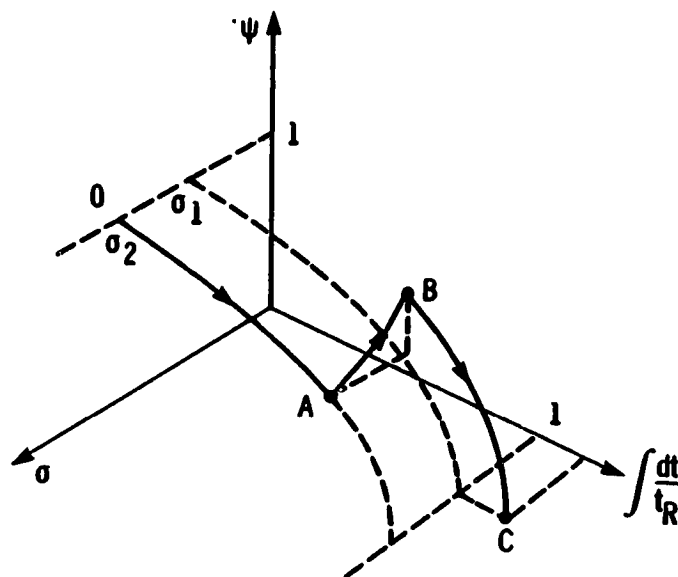


Figure 3. - Representation of step-down creep rupture test in $\left(\psi, \sigma, \int \frac{dt}{t_R}\right)$ space.

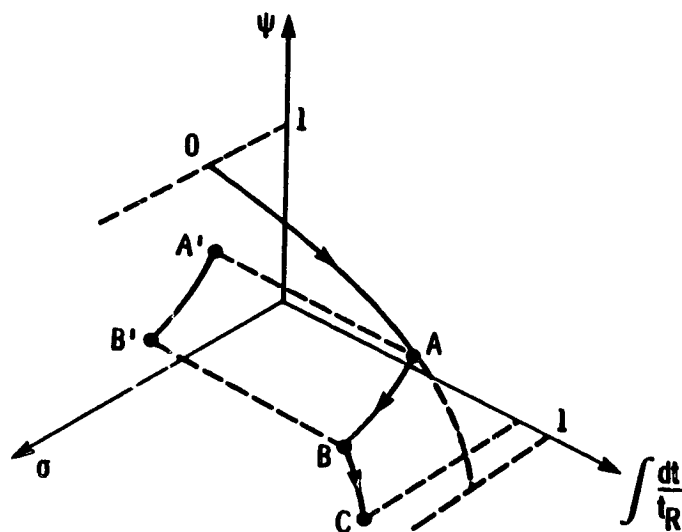


Figure 4. - Projection of path segment AB on σ, ψ plane.

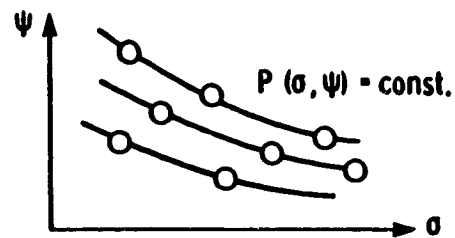


Figure 5. - Family of curves $p(\sigma, \psi) = \text{const.}$ in σ, ψ plane.

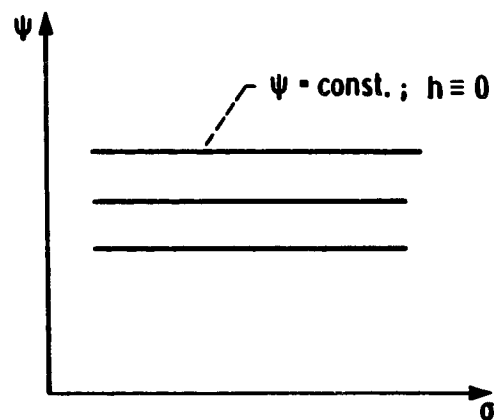


Figure 6. - Family of $\psi = \text{const.}$ curves in σ, ψ plane.

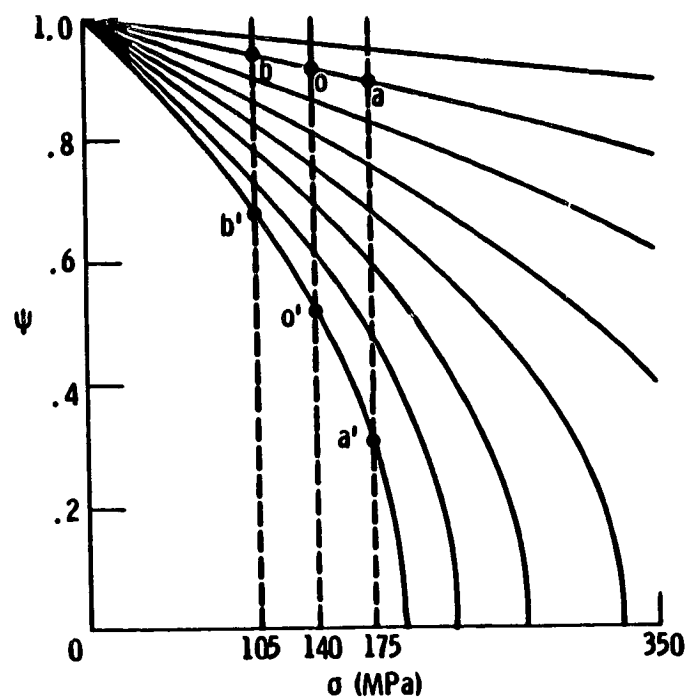


Figure 7. - Family of curves $p(\sigma, \psi) = \frac{1 - \psi^s}{\sigma} = \text{const.}$ in σ, ψ plane.

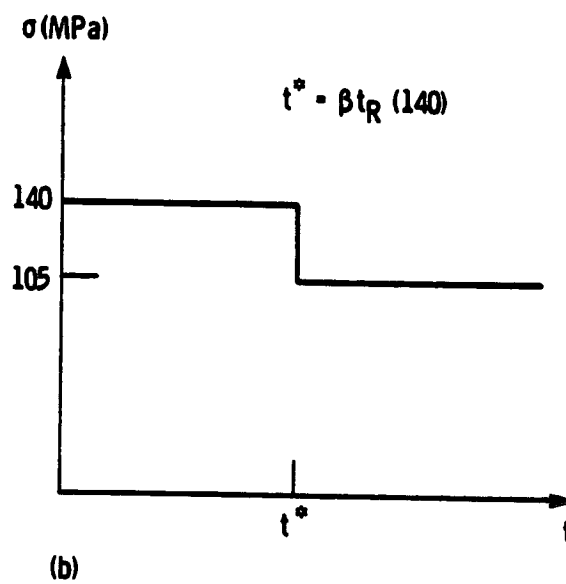
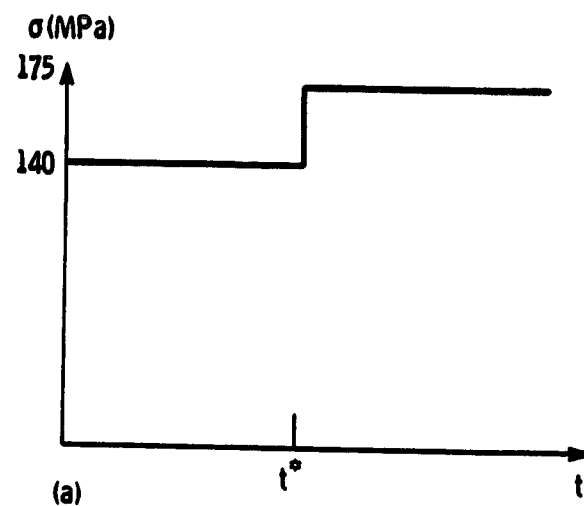


Figure 8. - Examples of step-up and step-down stress rupture tests.

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16. Abstract A creep damage accumulation model is presented that makes use of the Kachanov damage rate concept with a provision accounting for damage that results from a variable stress history. This is accomplished through the introduction of an additional term in the Kachanov rate equation that is linear in the stress rate. Specification of the material functions and parameters in the model requires two types of tests constituting a data base: (1) standard constant-stress creep rupture tests, and (2) a sequence of two-step stress creep rupture tests.					
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